Mean Time Between Failure

by Clive Roberts

The Mean Time Between Failure (MTBF) is a requirement you may find in various CCTV tender specifications. This is a grey area for many in CCTV, mostly because it is based on an empirically calculated expected lifetime which involves a little bit of statistical mathematics. For a successful and realistic calculation it is required that you obtain some data from various component manufacturers. Although MTBF can be calculated for each element in your CCTV system, such as cameras monitors, etc, nowadays we would be mostly concerned with the DVR MTBF, since majority of CCTV systems would rely heavily on DVRs life expectancy.
In this article, Clive Roberts explains how this is done.

Reliability of a system

Reliability $R(t)$ may be defined as the probability that a system will accomplish its designated mission in a satisfactory manner for a given time period when used under specified operating conditions.

$$R(t) = 1 - F(t) = \int_t^{\infty} f(t) \cdot dt \qquad (1)$$

$$f(t) = \frac{1}{\vartheta} e^{\frac{-t}{\vartheta}} \qquad (2)$$

$$R(t) = \int_t^{\infty} \frac{1}{\vartheta} e^{\frac{-t}{\vartheta}} = e^{\frac{-t}{\vartheta}} \qquad (3)$$

where ϑ is the mean life and t is the time period of interest.

Mean life, ϑ, is the average life time of the system under consideration, which for the exponential function is the Mean Time Between Failure (MTBF).

$$R(t) = e^{\frac{-t}{M}} = e^{-\lambda t} \qquad (4)$$

where λ is the failure rate and M is the MTBF.

Mean life and failure rate are related by…

$$\lambda = \frac{1}{\vartheta} \qquad (5)$$

The rate at which failures occur is a specified time interval called the failure rate for that interval. The failure rate per hour is given by…

$$\lambda = \frac{\text{number of failures}}{\text{total operating hours}} \qquad (6)$$

The failure rate can be expressed in terms of failures per hour or percentages of failures per period time (e.g. 10^3 or 10^6 hours).

Example 1

If 12 systems are tested for 900 hours and System A failed after 700 hours, System B failed after 450 hours, System C failed after 90 hours, System D failed after 560 hours and System E failed after 320 hours. There were five failures and a total operating time of 8420 hours. The failure rate can be calculated thus…

$$\lambda = \frac{5}{(7\cdot 900)+700+450+90+560+320} = \frac{5}{8420} = 0.0005938$$

The Mean Time Between Failure would be calculated as inverse value of λ, i.e.

$$MTBF = \lambda^{-1} = 1684\ hours.$$

This value may initially seem confusing, but don't forget that out of 12 systems we have only 5 failed within specified 900 hours, so we should expect that the average expected life time is longer than what is to be concluded from just the failed systems.

Example 2

If the operating cycle for a system is 780 hours and during that time the system in question fails 5 times. The first time the system takes 3 hours to repair, the second and third time it takes 5 hours to repair and the fourth and fifth time it takes 10 hours to repair, then…

$$\lambda = \frac{5}{780-3-5-5-10-10} = 0.006693$$

and the downtime for the system would be 33 hours.

The Mean Time To Repair (MTTR) can also be calculated. In this example the MTTR is 6.6 hours. Assuming an exponential distribution, the MTBF can be calculated ασ λ^{-1}. Therefore the MTBF is 149.4 hours. If the system is operated for 10 hours, the reliability is given as…

$$R(t) = e^{-0.006693\bullet 10} = 0.935 = 93.5\ \%$$

Up until now we have assumed an exponential distribution.

However, in reality, when a system is first put into service there are usually a higher than average number of failures – this is known as the **debugging period**.

Similarly, when a system reaches a certain age, the **wear-out period** begins.

This is when the system reaches its end of life period. This type of failure-rate curve is known as a bathtub curve.

The specific dynamics of this curve depend upon the system in question; each will have its own unique profile. If a system is modified or maintained during its life cycle then this will have an impact on the failure-rate curve. In software systems, failures are more likely to be associated with time, processor performance, amount of data to be processed, etc. and different failure rate curves can be observed.

The Effects of Component Reliability

In systems engineering, often a number of subsystems and/or components are combined to form a system each having their own reliability characteristics.

In a series network the reliability of a system is given as:

$$R_S = \prod_{n=1}^{n} R_n \qquad (7)$$

In plain English this means the total system reliability of a series of subsystems is given as the product of the subsystems reliability. This is a typical non-reduntant system as used in majority of CCTV installations.

Example 3

Consider a three component series system which is expected to operate for 300 hours.

The three systems have the following

MTBF: A=1000 hours; B=9000 hours; C=5000 hours

λ_A = 1/1000 = 0.001 failures / hour

λ_B = 1/9000 = 0.00011 failures / hour

λ_C = 1/5000 = 0.0002 failure / hour

$$R = e^{-\lambda_A t} \bullet e^{-\lambda_B t} \bullet e^{-\lambda_C t} = e^{-(\lambda_A+\lambda_B+\lambda_C)t} = e^{-(0.001+0.0001+0.0002)\bullet 300} = 0.675$$

This means that the probability of the system surviving 300 hours is 67.5%.

If the operational time were to be decreased to 100 hours the probability would increase to 87.7%.

CCTV Example

Consider a DVR running on Windows 2000 which has an MTBF of 72 weeks (which is 12,096 hours, as quoted by the independent consulting group NSTL), uses one hard disk (quoted by the manufacturer to have 300,000 hours MTBF) and is running on an off the shelf computer power supply (which has manufacturers MTBF of 50,000 hours at 25ºC). Thus:

MTBF: OS=12096 hours; HD=300,000 hours; PS=50,000 hours

λ_{OS} = 1/12,096 = 0.0000828 failures / hour

λ_{HD} = 1/300,000 = $3.3\ 10^{-6}$ failures / hour

λ_{PS} = 1/50,000 = 0.00002 failures / hour

The probability of system running for one year (8760 hours) is:

$$R = e^{-(0.0000828\ +\ 0.000003333+0.00002)\bullet 8760} = e^{-0.93} = 0.394 = 39.4\%$$

This means the probability of system running for one year is 39.4%.

Identical parallel redundant systems are used primarily to improve system reliability.

This would be the case when redundant disks are used in DVRs in RAID 5 configuration, or when a matrix switcher is made up of a hot swappable stand by switcher.

Assuming that both the parallel components are identical the system will function if **either**

A or B (or both) are working.

$$R_P = 1 - (1 - R)^n \qquad (8)$$

Example 5

A system which consists of two identical subsystems in parallel is to be considered, each with a reliability of 95%. The overall reliability is thus:

$R = 1 - (1 - 0.95)^2 = 0.9975$, or increased from 95% to 99.75%.

If a third identical system is added in parallel then the reliability becomes…

$$R = 1 - (1 - 0.95)^3 = 0.99875$$

If the parallel systems are non-identical then the following formula applies:

$$R_P = 1 - (1 - R_A)\cdot(1 - R_B)\ldots.(1 - R_n) \qquad (9)$$

It should be noted that hybrid approaches of series and parallel systems are often encountered.

When determining the reliability of standby systems, the Poisson distribution is used to calculate this because the standby systems display the constant λ_t characteristic.

The Poisson distribution is a discrete distribution which is applicable when the opportunity for the occurrence of an event is large, but when the actual occurrence is unlikely. The probability of x occurrences of an event of probability p in a sample n is:

$$P(x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!} \qquad 0 \le x \le \infty \qquad (10)$$

where the mean and variance of the distribution are equal and given by λ_t.

Example 5

Suppose that the system reliability for a configuration consisting of one operating system and two identical standby systems for a period of 100 hours is to be considered. The failure rate of each system is 0.008 failures per hour.

$$R = e^{-\lambda t} + (\lambda t)e^{-\lambda t} + \frac{(\lambda t)^2 e^{-\lambda t}}{2!}$$

$$R = e^{-0.8} + 0.8e^{-0.8} + 0.32e^{-0.8} = 2.12e^{-0.8} = 0.9526$$

Other Measures

Mean Time Between Maintenance can be broken into two facets – $MTBM_U$ (unplanned) and $MTBM_S$ (scheduled).

Availability can either be expressed as

A_i = Inherent availability;

A_a = Achieved availability or

A_O = Operational availability.

The inherent availability is the probability that a system or component, when used under stated conditions in an ideal support environment will operate satisfactorily at any point in time as required. The inherent availability does not take into account preventative or scheduled maintenance tasks or associated lead times (logistics, admin, etc.).

$$A_i = \frac{MTBF}{MTBF + MTTR}$$

where MTTR is the mean time to repair (or corrective maintenance time, Mct).

The achieved availability is the probability that a system or equipment, when used under stated conditions in an ideal support environment will operate satisfactorily at any point in time. Here, preventative and maintenance is considered. However, associated lead times are not.

$$A_a = \frac{MTBM}{MTBM + \overline{M}}$$

where M is the mean active maintenance time.

The operational availability is the probability that a system, when used under stated conditions in an actual operational environment, will operate satisfactorily when called upon.

$$A_O = \frac{MTBM}{MTBM + MDT}$$

where MDT is the mean maintenance down time. The reciprocal of MTBM is the frequency of maintenance. MDT includes active maintenance down time (M), and associated lead times.

Bibliography

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